MATHEMATICS

Paper 9709/01

Paper 1

General comments

There was a wide spread of performance evident in the scripts this year, but generally the standard remained high. There was a high proportion of excellent scripts and the majority of candidates found most questions accessible. At the lower end, it was pleasing to see fewer poor scripts from candidates who should really not have been entered for the examination. There was no evidence that candidates had had insufficient time and the standard of presentation was good.

Comments on specific questions

Question 1

For most candidates, this was a source of high marks. It was rare to see the binomial coefficient omitted and apart from the error of taking $\left(\frac{2}{x}\right)^2 = \frac{2}{x^2}$, most candidates obtained the correct answer. The only other common error was to misread the coefficient of x^2 as $\frac{1}{x^2}$.

Answer: 60.

Question 2

Most candidates realised that $\sin x = \frac{2}{5}$ and used either the identity $\sin^2 x + \cos^2 x = 1$ or constructed a 90º triangle with opposite 2 and hypotenuse 5 to evaluate the exact value of $\cos^2 x$. Similar methods led to the value of $\tan^2 x$ in part **(ii)**. There were however a significant number of candidates who fail to realise that 'exact' eliminates the decimal solution obtained by using a calculator. Finishing with a decimal answer that is not exact meant that the final accuracy marks could not be obtained.

Answer: **(i)** $\frac{21}{25}$; **(ii)** $\frac{4}{21}$.

Question 3

It was pleasing to see a large number of perfectly correct answers. Evaluation of the area of the sector was almost always correct, but unfortunately many candidates then assumed that *OBDC* was a square and took *OC* to be 12 cm. Many others used trigonometry accurately but evaluated *OC* as 10.39 cm, thereby losing accuracy marks at the end in attempting to return to $a\sqrt{3}$, where *a* is integral. Candidates must realise that this type of question is testing the syllabus item of knowing the exact value of sin 60º etc and that decimal equivalents are not going to lead to exact values.

Answer: $a = 54$, $b = 24$.

Question 4

This was very well answered. Part **(i)** was nearly always correct apart from careless numerical slips or obtaining the obtuse angle by considering $\overrightarrow{AO}.\overrightarrow{OB}$ instead of $\overrightarrow{OA}.\overrightarrow{OB}$. In part **(ii)**, candidates accurately obtained $\overrightarrow{AB}$ from **b** – **a** and usually obtained a correct expression for $\overrightarrow{AC}$. Many candidates incorrectly assumed that $\overrightarrow{AC}$ and $\overrightarrow{OC}$ were the same vector. The more serious problem arose over the evaluation of the unit vector for a majority of attempts showed no understanding of the meaning of a unit vector.

Answer: **(i)** 36.7°; **(ii)** $\frac{1}{3}\mathbf{i} - \frac{2}{3}\mathbf{j} + \frac{2}{3}\mathbf{k}$.

Question 5

This question was very well answered and a high proportion of candidates obtained full marks. Candidates realised the need to find the gradient of *AB* and the corresponding gradient of *CD* before finding the equation of *CD*. The solution of two simultaneous equations to find the coordinates of *D* in part **(ii)** presented candidates with few problems.

Answer: **(i)** $2y + 3x = 48$; **(ii)** *D*(10, 9).

Question 6

Part **(a)** proved to be surprisingly difficult. Most candidates recognised the situation as an arithmetic progression, but found it difficult to obtain either the first term (105), the last term (399) or, in particular, the number of terms. A minority of candidates realised that there were 43 terms in the progression. Part **(b)** presented fewer problems and the majority of candidates gave perfectly correct solutions. Methods varied from finding r first from $a = 144$ and $ar^2 = 64$, or by finding x first from $\frac{x}{144} = \frac{64}{x}$. Use of the formula for the sum to infinity was nearly always correct. Specific errors were to calculate r as 1.5 or as 0.67, the latter leading to loss of accuracy for the final answer mark.

Answer: **(a)** 10 836; **(b)(i)** 96, **(ii)** 432.

Question 7

This question was well answered, particularly part **(ii)**. In part **(i)** candidates recognised the need to differentiate though several went wrong in expanding the brackets. Surprisingly only about a half of all candidates realised the need to find the equations of the two tangents and to use simultaneous equations. The integration in part **(ii)** was sound and the majority of candidates were able to show that the two regions had the same area.

Answer: **(i)** $1\frac{2}{3}$.

Question 8

Although the majority of candidates showed a good understanding of the need to use function of a function to obtain the differential in part **(i)**, there were still a significant number who omitted to multiply by −2, the differential of 5 − 2*x*. Part **(ii)** proved to be too difficult for about half the candidates, many of whom incorrectly assumed that $\frac{dx}{dt}$ was equivalent to $\frac{dy}{dx}$ multiplied by $\frac{dy}{dt}$. In part **(ii)**, candidates on the whole showed considerable maturity in the way they integrated $\frac{36}{(5-2x)^2}$, though some attempted to expand the denominator, others omitted to divide by −2 and others obtained $(5 - 2x)^{-3}$ instead of $(5 - 2x)^{-1}$. The use of limits was generally good, though a minority of attempts automatically assumed that the lower limit of 0 could be ignored.

Answer: **(i)** $1\frac{1}{3}$; **(ii)** 0.015 units per second.

Question 9

Parts **(i)** and **(ii)** proved to be too difficult for at least a half of all candidates, though there were many attempts at fiddling the answers. Candidates realising that the height of the vertical end pieces was $3x$ (usually by Pythagoras) fared better, though it was common to see the area of the open top included in the total surface area. Use of 'volume = cross-sectional area $\times$ length' in part **(ii)** was not well known. Parts **(iii)** and **(iv)** were more accessible and tended to be correct. The differentiation and solution of $\frac{\mathrm{d}V}{\mathrm{d}x} = 0$ was accurate and candidates had little trouble in deducing that the volume was a maximum, almost always by considering the sign of the second differential.

Answer: **(iii)** $1\frac{2}{3}$; **(iv)** Maximum.

Question 10

Candidates had obviously been well taught in the basic skills required for parts **(i)** and **(ii)**. Although part **(i)** was usually correct, some candidates solved the equation $x^2 - 3x = 4$ as $x(x-3) = 4 \Rightarrow x = 4$ or $x = 7$.

It was rare for candidates to give the incorrect range of $-1 < x < 4$. Part **(ii)** was very well answered. Part **(iii)** was poorly answered with a large proportion of candidates failing to appreciate that the minimum value of $(x-a)^2 - b$ is $-b$ and the range of f is $\geqslant -b$. There were some interesting answers to part **(iv)** with a minority of attempts stating that the quadratic function f was not one-one for real x. Part **(v)** presented candidates with difficulty when they attempted to solve the equation $x - 3\sqrt{x} = 10$ Very few recognised this as a quadratic equation in $\sqrt{x}$. Many candidates squared both sides incorrectly to form the equation $x^2 + 9x = 100$ or $x^2 - 9x = 100$. Better attempts isolated the $3\sqrt{x}$ prior to squaring, though these candidates rarely discarded the spurious solution of $x = 4$.

Answer: **(i)** $x < -1$ and $x > 4$; **(ii)** $a = 1\frac{1}{2}$, $b = 2\frac{1}{4}$; **(iii)** $\mathrm{f}(x) \geqslant -2\frac{1}{4}$; **(iv)** no inverse, f not one-one; **(v)** $x = 25$.

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Paper 2

General comments

As usual, there was a wide range of candidate performance. Some candidates displayed little or no knowledge of the key techniques of the calculus, differentiation and integration rules and results, and as a result were completely out of their depth. A number of key topics unfortunately proved problematic for most candidates, namely **Question 5(i)**, **Question 7** and **Question 8**. Degrees instead of radians were frequently adopted in **Question 5(iii)**. Candidates showed no sign of running out of time. The Examiners were impressed by the many candidates who had clearly been well prepared and who showed considerable finesse in much of what they did. Weaker candidates often presented their work badly.

Comments on individual questions.

Question 1

Almost all candidates opted to square each side of the given inequality and to then solve the resultant quadratic equation or inequality. On the left-hand side, $4x^2$ was often erroneously replaced by $2x^2$ and, more worryingly, many candidates squared only the left-hand side of the initial inequality. The last mark was very frequently lost by candidates adopting incorrect inequality signs. A good guide as to where the solution is valid is to try the simplest of all values, $x = 0$; if it firs the given inequality, then the value $x = 0$ must belong to the final solution range, and vice-versa. A few other candidates in **Question 1** simply found $x = 1$ was a key value

Answer: $x < \frac{1}{3}$, $x > 1$.

Question 2

(i) A minority of the candidates realised that $4^x = (2^x)^2 = y^2$. Values such as $2y$ or 2^{x^2} were common.

(ii) Even those who were incapable of tackling part **(i)** were able to set up and solve correctly a quadratic equation in $y = 2^x$, Sadly, many failed to then find the corresponding values for x.

Answer: **(i)** $4^x = y^2$; **(ii)** ± 1.58

Question 3

(i) This part posed few problems, bar a few solutions involving setting $f\left(-\frac{3}{2}\right) = 0$.

(ii) Many candidates at no stage commented that $x = \frac{3}{2}$ was a solution of the equation, not just that $(2x - 3)$ was a factor of the cubic expression. Most successfully divided the cubic by $(2x - 3)$ to get a second factor $(2x^2 + 3x + 1)$ and, bar the occasional sign error, deduced the two further roots arising from $2x^2 + 3x + 1 = 0$.

Answer: **(ii)** $x = \frac{3}{2}, -1, -\frac{1}{2}$.

Question 4

(i) A number of candidates never assigned a value to tan 45° or gave it an erroneous value. Much more serious was the large number who adopted a false rule $\tan(A \pm B) \equiv \tan A \pm \tan B$ in their initial step. After excellent initial expansions of use two tangent terms on the left-hand side; about half of these candidates could not successfully proceed further. Sign errors abounded and the common denominator was lost during the process of merging the two terms.

(ii) This was extremely well done even by weaker candidates. A few found only the first quadrant solution and others wasted time repeating their analysis of part **(i)**, and some candidates stated $x = \tan^{-1}\left(\frac{1}{2}\right)$.

Answer: **(ii)** x = 22.5°, 112.5°.

Question 5

(i) Here, for the first time in the paper, general confusion was evident. A few candidates gave one or two of the 3 terms correctly, but rarely a 3-term equation. There were even examples such as $2\pi r^2$ for the area of a circle, or $\frac{1}{2}r^2\alpha$ as the area of segment *OAB*. Very few could equate $\frac{1}{6}\pi r^2$ with the difference between $\frac{1}{2}r^2\alpha$ and $\frac{1}{2}r^2\sin\alpha$.

(ii) Instead of (correctly) finding the signs of $\pm\left(x - \sin x - \frac{1}{3}\pi\right)$ at $x = \frac{1}{2}\pi$ and $\frac{1}{3}\pi$, a falsely defined function $\left(\frac{1}{3}\pi + \sin x\right)$ was utilised. Very few candidates correctly evaluated $\mathrm{f}\left(\frac{\pi}{2}\right)$ and $\mathrm{f}\left(\frac{\pi}{3}\right)$, and drew a correct conclusion.

(iii) Despite the fact that only radian measure had been referred to in the question, around half of all candidates adopted degrees for their angles in part **(iii)**. Among those who correctly worked in radians, many did not perform sufficient, i.e. at least 4, iterations and drew conclusions too promptly. Others worked immaculately, but failed to round their solutions to 2 decimal places.

Answer: **(iii)** 1.97

Question 6

(i) There was some poor differentiation, e.g. a single term derivative or false versions of the result for differentiating a quotient (or product). Those who correctly found $\frac{\mathrm{d}y}{\mathrm{d}x}$ invariably went on to score full marks.

(ii) It was surprising to see 3 or even 4 intervals used, with many false values of the interval, h. This was an easy question of its type, but candidates were weaker then usual in tackling it. Even one interval was a popular choice.

(iii) Few candidates gave a convincing reason for an over-estimate, and almost all had only one strip and line joining x = 1, y = 7.39 to x = 2, y = 27.3, i.e. one trapezium was deemed adequate.

Answer: **(i)** (0.5, 5.44); **(ii)** 15.4; **(iii)** over-estimate.

Question 7

(i) Many candidates used the chain rule correctly, but many settled for a derivative $\sec^2 2x$ or $\sec^2 x$.

(ii) The indefinite integral was often given as $\tan 2x$ or $2\tan 2x$ or a function bearing no relation to any tangent function. Very few candidates used $\tan^2 2x \equiv \sec^2 2x - 1$ and a number of totally perverse forms such as $\frac{1}{2}\tan^2 2x$ or $\frac{\tan^2 x}{2\sec x \tan^2 x}$ were seen, plus many solutions based on $\tan^2 x = \frac{\sin^2 x}{\cos^2 x}$ with strange combinations of sines and cosines in incorrect solutions.

(iv) Nearly all candidates failed to succeed in getting $\int \frac{dx}{2\cos^2 x}$ correctly, but then set this equal to $2\int \sec^2 x \, dx$, etc. A minority simply had no idea what to do, with solutions such as $\ln(1+\cos 4x)$ or $\frac{1}{2+2\cos^2 2x}$ as common forms of the indefinite integral and the first step. Others tried to write the integrand in terms of sin x and cos x.

This question has a natural flow of ideas running through it, but only the very best candidates could see such a sequence of calculations.

Answer: **(i)** $2\sec^2 2x$; **(ii)** $\frac{1}{2}\sqrt{3} - \frac{1}{6}\pi$; **(iii)** $\frac{1}{4}\sqrt{3}$.

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Paper 3

General comments

The standard of work by candidates varied greatly and resulted in a wide spread of marks. Candidates seemed to have sufficient time and no question appeared to be unduly difficult. The questions that were done particularly well were **Question 1** (inequality), **Question 8** (partial fractions) and **Question 10** (calculus and iteration). The questions that caused the most difficulty were **Question 4** (differential equation), **Question 5** (binomial series) and **Question 7** (vector geometry).

In general the presentation of work was good but there are two respects in which it was sometimes unsatisfactory. Firstly there were a few candidates who present their work in a double column format. This makes marking difficult for Examiners and it would be helpful if Centres could discourage the practice. Secondly, though the rubric for the paper informs candidates of 'the need for clear presentation in your answers', there are some who do not show sufficient steps or make clear the reasoning that leads to their answers. This occurs most frequently when they are working towards an answer given in the question paper, for example as in **Question 6(i)**. Examiners penalize the omission of essential working.

The detailed comments that follow draw attention to common errors and might lead to a cumulative impression of indifferent work on a difficult paper. In fact there were many scripts showing a very good understanding of all the topics being tested.

Where numerical and other answers are given after the comments on individual questions it should be understood that alternative forms are often acceptable and that the form given is not necessarily the sole 'correct answer'.

Comments on Individual Questions

Question 1

Most candidates answered this well. Some gave the end points of the final answer to 3 decimal places rather than to 3 significant figures.

Answer: $1.83 < x < 1.95$.

Question 2

The most common approach was to use the formula for tan $2x$ and, after some manipulation, obtain the quadratic equation $3\tan^2 x = 1$. However there were also successful solutions involving a quadratic equation in sin x or, less frequently, one in cos x. Apart from algebraic slips when trying to form the quadratic, the main error was to ignore or mishandle the negative root of the quadratic.

Answers: 30°, 150°.

Question 3

Though most candidates obtained the first derivative correctly and set it equal to zero, a substantial number were unable to solve the resulting equation. Errors of principle were made when taking logarithms or handling powers of e. The method of determining the nature of a stationary point by examining the sign of the second derivative at the point was popular and well understood.

Answers: **(i)** $\frac{1}{2}\ln 2$; **(ii)** maximum.

Question 4

Nearly all candidates knew that the first step in this question was to separate variables, yet a substantial number made serious errors at this stage. Those that separated correctly usually made a good attempt at integration and the evaluation of an arbitrary constant. The main mistakes were (i) omission of the factor $\frac{1}{2}$ when integrating $\frac{y}{1+y^2}$, (ii) omission of the arbitrary constant, and (iii) errors in exponentiation.

Answer: $y^2 = 5e^{2x} - 1$.

Question 5

(i) This was poorly answered on the whole. The initial simplification defeated many candidates. Even when it was correctly done and the given expression was reduced to $2x$, some candidates failed to give enough working to show how the final relation could be deduced.

(ii) In this part the given expression is the reciprocal of a sum. Examiners were surprised to find that there were many candidates who believed it to be equivalent to the sum of the reciprocals, and thus expanded the expression $(1+x)^{-\frac{1}{2}} + (1-x)^{-\frac{1}{2}}$.

The most popular correct approach was to expand the numerator of the right hand side of the relation given in part **(i)** and divide by $2x$. A common error was to expand only as far as the terms in x^2. Those that did expand as far as the terms in x^3 sometimes made sign errors in the coefficients of these terms or in their subtraction. A less common method was to obtain a series expansion of the denominator of the given expression and find its reciprocal by long division. Many of these attempts reached the expression $2 - \frac{1}{4}x^2$ but went no further.

Answer: **(ii)** $\frac{1}{2} + \frac{1}{16}x^2$.

Question 6

The majority of candidates understood how to obtain the first derivative from an implicit equation and the first part was quite well answered. Attempts at the second part usually began with the recognition that $y = x^2$, though some candidates thought that $\frac{dy}{dx} = 1$, and others that $2y^2 = x$. Examiners encountered numerous errors in the algebra that immediately followed, for example in finding the x-coordinate by solving $x^3 + 2(x^2)^3 = 3x^3$. Also, having found one coordinate, e.g. $x = 1$, some candidates substituted in the original curve equation in order to find the other coordinate. This leads to three values and further work is needed to deduce the correct solution to the problem.

Answer: **(ii)** (1, 1).

Question 7

(i) A number of successful methods were seen. One approach showed that the coordinates of the general point of the line always satisfy the plane equation. A second showed that the line is parallel to the plane and that one point on the line, usually (0, 1, 1) also lies in the plane. A third verified that two points on the line lie in the plane. Some candidates believed that it was sufficient to show that one point on the line lies in the plane, and others thought that it sufficed to show that the line was perpendicular to the normal of the plane.

(ii) The usual approach was to find a vector normal to the new plane and use the point (2, 1, 4) to find an equation for the plane. Successful candidates, realising that the new normal is perpendicular to both the line and to the normal of the original plane, used a pair of simultaneous equations or a vector product to find a set of values for a, b, c. However Examiners often found candidates using an inappropriate vector when setting up the equations or the product. Also some candidates incorrectly assumed the new normal vector to be $\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$ or $\mathbf{i} - 2\mathbf{j} + \mathbf{k}$.

Answer: **(ii)** $4x + y - 2z = 1$.

Question 8

Both parts of this question were generally very well answered. Nearly all candidates, using an appropriate form of partial fractions, formed an expression identically equal to $7x + 4$, and had a sound method for evaluating the constants. Most of the errors occurred in the formation of the identity. A careful check at this stage might have prevented some candidates from losing several marks in part **(i)**. The integration was usually well done apart from slight slips, though some candidates thought the integral of $\dfrac{C}{(x+1)^2}$ involved a logarithm. Most candidates gave sufficient working to show that both the limits had been substituted properly.

Answer: **(i)** $\dfrac{2}{2x+1} - \dfrac{1}{x+1} + \dfrac{3}{(x+1)^2}$.

Question 9

The first two parts were done well. In part **(iii)** most candidates could plot the point representing u and realised that a circle of radius 1 was needed. However when u had been found and the associated point plotted correctly, the circle was quite often centred at the point representing $-1 - \mathrm{i}$. Part **(iv)** discriminated well and only a few candidates could calculate the minimum of $|z|$.

Answers: **(i)** $1 + \mathrm{i}$; **(ii)** $\sqrt{2}$, 45°; **(iv)** $\sqrt{2} - 1$.

Question 10

There were many correct solutions to part **(i)**. In part **(ii)** some candidates started the iteration with their calculators in degree mode and persisted with the iteration in spite of the fact that convergence was very slow. Also a minority could not interpret $\dfrac{1}{2}\tan^{-1}\left(\dfrac{1}{2x}\right)$ correctly. However the majority were successful, though some forgot to round their final answer to 2 decimal places. Part **(iii)** was generally well answered. Nearly all candidates tried to apply the method of integration by parts correctly. Most of the errors arose in the integration of $\cos 2x$ and $\sin 2x$.

Answers: **(ii)** 0.43; **(iii)** $\dfrac{1}{8}(\pi - 2)$.

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Paper 4

General comments

The paper was generally well attempted with a substantial number of candidates demonstrating a clear understanding of the topics examined. As usual there was a significant number of candidates who scored very low marks, and who were clearly not ready for examination at this level. There was no question in the paper that seemed to cause particular difficulty and no question that was found to be especially easy.

Problems of accuracy were much more widespread at this sitting than previously. The three main roots of error were truncation after 3 significant figures instead of rounding, premature approximation, and giving answers to 2 significant figures. The requirement of the rubric is that numerical answers should be given correct to 3 significant figures. The exceptions to this are where answers are exact, and where answers are in degrees (for which answers correct to 1 decimal place are required).

To achieve answers correct to 3 significant figures candidates should be aware of the need to use intermediate values to a sufficient degree of accuracy, which in some cases may exceed 3 significant figures. This need not cause difficulty if candidates make use of their calculators with an appropriate level of skill. Examiners have a procedure in place to avoid over penalising candidates who repeatedly use premature approximation. However it would benefit both candidates and Examiners if candidates gave answers correct to 3 significant figures.

Answers to 2 significant figures that were each given by many candidates include 590 in **Question 1(i)**, 0.67 in **Question 2(ii)**, 0.86 in **Question 3(ii)**, 27 in **Question 6(i)**, 0.15 in **Question 7(ii)** and 6.6 in **Question 7(iii)**. As in the case of premature approximation, Examiners have a procedure in place to avoid over penalising candidates who repeatedly infringe the rubric, but it would benefit candidates if they gave their answers to the required accuracy.

Comments on specific questions

Question 1

(i) Although many candidates answered this correctly, there were several wrong answers that were fairly common. These included answers obtained from 30 x 20, 30 x 20sin10°, 30 x 20cos15°, 30 x 20cos25° and 30 x 20cos10° – 8 x 10sin15°.

(ii) This was the best attempted of the three parts.

(iii) A substantial proportion of candidates who scored full marks in parts **(i)** and **(ii)**, failed to see the connection between their answers and this part of the question. A significant minority of candidates who failed to answer part **(i)** correctly, nevertheless obtained the correct answer by starting afresh in this part of the question.

Answers: **(i)** 591 J; **(ii)** 414 J; **(iii)** 177 J.

Question 2

(i) Many candidates assumed that the frictional force acts downwards and thus obtained $F = -1.67$. Most such candidates satisfactorily reconciled the minus sign with error in their assumption, although some left the answer as –1.67 N, or simply wrote $F = -1.67 = 1.67$ N.

(ii) This part was very poorly attempted and most candidates failed to recognise that the normal force acts horizontally. The formula $F = \mu R$ was well known to candidates, but as well as having an incorrect value for R many candidates eschewed the given value for F in favour of a value other than 1.67.

Answer: **(ii)** 0.668.

Question 3

(i) This part of the question was very well attempted.

However some candidates failed to recognise that what was required is an *instantaneous* acceleration. Such candidates obtained the kinetic energy (937.5 J) required to increase speed from zero to 5 ms^{-1}. They then assumed the work done by the cyclist is equal to the increase in kinetic energy and deduced that the time taken is $937.5 \div 420 = 2.232\ldots$seconds, and hence by assuming the acceleration is constant (notwithstanding the constant power), obtained $a = (5 - 0) \div 2.232.. = 2.24$ ms^{-2}.

(ii) This part of the question was less well attempted, although there were many correct answers. The most common errors included the omission of the forward force of 84 N, attaching the factor $\cos 1.5°$ to this forward force, and misreading 1.5° as 15°.

Answers: **(i)** 1.12 ms^{-2}; **(ii)** 0.858 ms^{-2}.

Question 4

(i) Almost all candidates obtained the correct expression for $a(t)$. However many candidates did not understand the term 'initial'. A very common error was to find the acceleration when the velocity is zero.

(ii) This part of the question was very well attempted.

Answers: **(i)** 1.25 ms^{-2}; **(ii)** 61.2 m.

Question 5

Very many candidates implicitly treated the question as one of vertical motion in a straight line, making repeated use of the formula $v^2 = u^2 + 2gs$. Candidates using this approach must explain very carefully why it yields correct answers in these very different circumstances, in order to score any marks. This is of course profoundly more difficult than making use of the principle of conservation of energy and, not surprisingly, no explanations were seen by Examiners.

(i) A surprisingly large proportion of candidates thought the greatest speed occurs at N and thus obtained an answer of 5 ms^{-1} or 8.54 ms^{-1} (using $\Delta h = 2.45 + 1.2$).

(ii) In this part of the question many candidates thoughtlessly applied 'PE = KE', each of KE and PE being an instantaneous quantity, thus obtaining the answer 6 J. Candidates must be aware that the simple formula applies to *changes* in PE and KE during some time interval.

(iii) This part of the question was generally poorly attempted or omitted. Many candidates who obtained the answer 6 J in part **(ii)** obtained the relatively correct answer of 4.90 ms^{-1} in this part.

Answers: **(i)** 7 ms^{-1}; **(ii)** 6.25 J; **(iii)** 5.

Question 6

(i) Most candidates who used Pythagoras' theorem obtained the answer $P = 10$, but many found difficulty in finding the value of R. Many candidates found α and then used trigonometry to obtain P. A large proportion of such candidates made the error involving a minus sign, referred to in part **(ii)** below.

(ii) In finding the value of α many candidates used $\tan\alpha = \frac{9.6}{-2.8}$ or $\cos\alpha = \frac{-2.8}{10}$, but nevertheless obtained the answer $\alpha = 73.7$. Parts **(a)** and **(b)** were reasonably well attempted.

(iv) In very many cases this part was omitted or poorly attempted. It was common to see the required angle calculated as the angle between the direction of the force of magnitude 25 N, and the direction of the resultant of the two forces. Another common error revealed a widespread misunderstanding of the meaning of 'resultant'. Instead of finding the components of the resultant (and hence the value of θ), candidates treated the problem as one of *equilibrium* of *three* forces, the third force being that of magnitude R N.

Answers: **(i)** 10, 26.9; **(ii)(a)** 24 N, **(b)** 7 N; **(iii)** 38.1.

Question 7

(i) Most candidates obtained R = 9.336 m correctly. However there were few completely correct solutions leading to F = 1.416 m. Most candidates found the weight component $mg\sin21°$, but very many just noted that subtraction from 5 m yielded the correct answer. Such candidates gave no indication of an understanding of where the '5' comes from.

Among the candidates who found the acceleration using $v = u + at$, almost all obtained a = 5 ms^{-2} for the upward acceleration instead of –5 ms^{-2}. In applying Newton's second law signs were adjusted (without explanation) to accommodate the error and produce the given answer.

(ii) Almost all candidates used $\mu = \frac{F}{R}$, but a large proportion gave the truncated answer 0.151 or the 2 significant figure answer 0.15.

(iii) Most candidates who attempted this part of the question recognised the need to find the relevant distance. Although 10 m was the most usual value found, 20 m and 30 m were also very common. Only a minority of candidates found the acceleration correctly, there being very many candidates who did not make an attempt to do so. Common wrong values for the acceleration include 5 ms^{-2}, 10 ms^{-2} and 10sin21° ms^{-2}.

Candidates who had calculated both the distance and the acceleration were equipped to use $v^2 = 2as$, and most did so. However the use of $v = u + at$ was also common and in almost all such cases the value of t relating to the downward motion was not calculated. The value t = 2 for the upward motion was used instead.

Answers: **(ii)** 0.152; **(iii)** 6.58 ms^{-1}.

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Paper 5

General comments

This paper proved to be a fair test for any candidate with a clear understanding of basic mechanical ideas.

Good and average candidates had sufficient time to attempt all the questions on the paper.

It was pleasing that most candidates worked to three significant figures or better and that not many candidates used premature approximation.

Only a handful of candidates used g = 9.8 or 9.81.

The drawing of clear diagrams on the answer paper would be a helpful aid to candidates in presenting their work. Solutions to all questions, except **Question 7**, would have benefited from a clear diagram.

Questions 6 and **8** proved to be the harder questions.

Comments on Individual Questions

Question 1

This question was well done by the good and average candidates. The weaker candidates failed to recognise that they needed to substitute $\theta = 0$ into the trajectory equation quoted on the list of formulae. Some candidates ended up with $y = \frac{5}{64}x^2$ instead of $y = -\frac{5}{64}x^2$.

Part **(ii)** was well done by many candidates.

Answers: **(i)** $y = -\frac{5}{64}x^2$; **(ii)** 45 m.

Question 2

Candidates scored well on this question.

In part **(i)** some candidates used $a = r\omega^2$ to find a and then found v to calculate the horizontal component using $F = \frac{mv^2}{r}$, which was rather a long winded approach when $F = mr\omega^2$ could be used directly.

Part **(ii)** was well done.

Answers: **(i)** 0.6 N, 0.135 N.

Question 3

(i) This was generally well done.

(ii) Often $R = ma$ only was used and no component of the tension was seen.

A number of candidates mixed up sine and cosine and were only able to score the two method marks.

Answers: **(i)** 2.5 N; **(ii)** 1.22 N.

Question 4

This question was a good source of marks for the candidates, as many correct methods could be used. Unfortunately some of the candidates mixed up the various approaches and failed to produce the correct answer. A common mistake when using vertical motion was to consider g as positive. Sometimes the motion to the highest point was considered which produced half the range. The horizontal distance from the ground to A was then calculated. The two values were subtracted but unfortunately candidates sometimes failed to double the result.

Answer: 24 m.

Question 5

Quite a number of candidates used 3 as the weight instead of the mass.

(i) Some candidates could not take moments and ended up with one side of the equation as a moment and the other side as a force i.e. $3g \times 1.5 = T\cos 15°$. However, this part was well done by many candidates.

(ii) The tension at A was often taken to be the same as that at B. Good candidates usually produced perfect solutions.

Answers: **(i)** 18.6; **(ii)** 21.8° or 21.9°

Question 6

(i) The centre of mass of a circular sector is stated on the list of formulae. Some candidates used the wrong formula. Some common mistakes were to take $r = 5$ and $\alpha = \frac{\pi}{4}$. Only a handful of candidates managed to arrive at $2+\frac{2}{\pi}$.

(ii) Most candidates recognised that they had to use $A\bar{x} = A_1\bar{x}_1 + A_2\bar{x}_2$ and the correct answer appeared quite often. Some common mistakes seen were πr^2 instead of $\frac{\pi r^2}{2}$ for one of the areas and to take the centre of mass of the triangle as $\frac{8}{3}$ m instead of $\frac{4}{3}$ m from AB.

Question 7

This question was a good source of marks for many candidates. Weaker candidates tried to use the equations of rectilinear motion instead of integration.

(i) Many candidates used $\frac{\mathrm{d}v}{\mathrm{d}t} = v\frac{\mathrm{d}v}{\mathrm{d}x} = 0.6x^{0.2}$, then separated the variables and integrated. One of the careless errors that occurred was to get v^2 instead of $\frac{v^2}{2}$.

(ii) This part was well done by many candidates. A number went straight from $\int x^{-0.6}\mathrm{d}x = \int \mathrm{d}t$ to $t = 2.5x^{0.4}$. This is a given answer and so candidates should show $\dfrac{x^{0.4}}{0.4} = t$ or some other intermediate step.

(iii) Many candidates found the distance correctly.

Answers: **(i)** $v = x^{0.6}$; **(iii)** 32 m.

Question 8

A good clear diagram with all the relevant distances marked on it would have helped candidates to more clearly understand this question.

Quite a number of candidates used $v^2 = u^2 + 2gs$, completely ignoring any elastic energy.

(i) An energy equation was usually considered but too often sign errors occurred or the loss of potential energy was given as 1.8 not 0.6 J.

(ii) **(a)** An energy equation was set up but again too often sign errors occurred or the loss of potential energy was $0.2 \times 10(1.2 + x)$ or $0.2 \times 10(0.6 + x)$ instead of $0.2 \times 10(1.5 + x)$.

(b) Many candidates solved the equation to get 1.17 and either just left it as the answer or only added 0.6 or 0.9 instead of 1.5.

Answers: **(i)** 3.12 ms^{-1}; **(ii)(b)** 2.67 m

MATHEMATICS

Paper 9709/06

Paper 6

General comments

This paper produced a wide range of marks. There were many good attempts at the whole paper. Premature approximation leading to a loss of marks was only experienced in a few scripts, most candidates realising the necessity of working with, say, $\sqrt{21}$ instead of 4.58. Candidates seemed to have sufficient time to answer all the questions, and only the weaker candidates answered questions out of order.

Candidates were generally good on numerical aspects of the syllabus, less so on related terms, e.g. mutually exclusive, independent, exhaustive, grouped frequency table.

Comments on specific questions

Question 1

There seems to be a pattern that the first question, which is intended to be an easy one, usually manages to prove difficult for many candidates, and this year was no exception. Some candidates did not know what a grouped frequency table was, and produced stem-and-leaf diagrams. Some produced a histogram with the correct group boundaries. Some candidates gave 4 equal groups not 5, and some gave 5 unequal groups. Having negotiated that hurdle, there were many cases of wrong frequencies which did not even sum to 30. It was pleasing to see tally charts on good scripts.

Question 2

This was a straightforward question and was usually answered correctly. Several candidates however put $\Sigma xp = 1$ in part **(i)** rather than $\Sigma x = 1$ and thus lost both marks in part **(i)**. A follow through mark was given in part **(ii)** for $E(X)$ which then came to be 1. There were the usual few candidates who forgot to subtract $[E(X)]^2$ from $E(X^2)$ but apart from that, this question gave many candidates 5 easy marks.

Answers: **(i)** 0.15; **(ii)** 1.56, 1.41.

Question 3

Surprisingly, quite a few candidates did not appreciate that there were 60 minutes in an hour, and treated 3 hours 36 minutes as 3.36 hours. Others assumed symmetry and thought that the upper quartile – median = median – lower quartile. Yet another mistake was to divide the median (216 minutes) by 4 to get 54 minutes, which was the correct answer! A few candidates failed to use the required scale of 2 cm to represent 60 minutes. In part **(ii)**, many candidates were not able to read their scale correctly and put the median and quartiles in the wrong places.

Answers: **(i)** 54 minutes.

Question 4

Parts **(i)** and **(ii)** were basically well done. Most candidates gave a list or possibility space, but there were some who did not appear to understand the meaning of 'sum', difference', or 'product'. Part **(iii)** showed all too clearly that many candidates had very little idea of the meaning of the term 'mutually exclusive', mistaking it for independence or mutually exhaustive.

Answers: **(i)** $\frac{1}{3}$; **(ii)** $\frac{5}{9}$.

Question 5

Part **(i)** was often omitted. Definitions of a normal distribution were many and varied, some examples being ‘wristwatches’, ‘a whiteboard’, ‘cash in hand’, ‘to know the true area of land’, ‘chalk’, ‘the weather’. Several candidates tried to draw their example from questions in past papers which was acceptable provided they named a correct variable. For instance ‘fish in a river’ would not score any marks but ‘length of fish in a river’ would.

The standardised *z*-value in part **(ii)** was equally likely to be the incorrect +0.64 as the correct – 0.64. Thus not many candidates managed to score full marks in this part, and in part **(iii)** few candidates remembered to multiply their probability by 300, and even fewer corrected it to a whole number (how many observations).

Answers: **(ii)** 12.9; **(iii)** 7.

Question 6

Parts **(i)** and **(iii)** in this question were done surprisingly well with many candidates gaining full marks here. The most common error in part **(i)** was 6!   3! Part **(ii)** discriminated well between those candidates who had a rote learning of permutations and combinations and those who could think a bit harder. The 6! for the men was often seen, gaining a mark, but the 7   6   5 for the women was rarely seen, though sometimes a 7 was seen. The alternative method by subtraction produced some part marks but it was difficult to achieve full marks by this method.

Answers: **(i)** 362 880; **(ii)** 151 200; **(iii)** 64.

Question 7

In part **(i)** a pleasing number of candidates performed well although a significant minority failed to recognise the binomial distribution and others evaluated the correct answer, 0.117, only to double it, or square it. Part **(ii)** was generally well done by those candidates who recognised the binomial. Common errors included evaluating P(17), faulty arithmetic, premature rounding to 0.0036 or 0.004. Part **(iii)** was well done with fewer candidates failing to make a continuity correction than in the past. The continuity corrections were not always correct, but at least some method marks could be awarded. Some candidates used 60 for the mean and not 90, which meant that the final marks could not be awarded because the standardised z-values were too large.

Answers: **(i)** 0.117; **(ii)** 0.00361; **(iii)** 0.556.

MATHEMATICS

Paper 9709/07

Paper 7

Overall, this proved to be a fair and discriminating paper. Candidates were able to demonstrate and apply their knowledge on the topics examined. There was a good spread of marks, with only very few candidates who appeared to be totally unprepared for the examination.

There were some questions that many candidates found demanding. In particular, **Questions 5**, **2(ii)**, **7(iv)** and **(v)**, and **6** caused some problems, which are detailed below. Questions on continuous random variables (**Question 7**) have, in the past, been particularly well attempted. However, on this paper part of the question was less 'routine' and candidates did not score quite so well, often gaining very few of the last 5 marks. **Questions 1** and **6** involved setting up a null and alternative hypothesis. Examiners noted that many candidates had calculations following a hypothesis that did not match their intentions.

In general, work was well presented with methods and working clearly shown.

Some marks were lost by candidates due to premature approximation and inability to round successfully answers to three significant figures. This has been mentioned in detail on many occasions in the past and continues to be a cause of loss of marks.

Lack of time did not appear to be a problem, with most candidates offering solutions to all questions.

Detailed comments on individual questions now follows, though it should be noted that whilst the comments indicate particular errors and misconceptions, there were also many very good and complete answers to each question.

Question 1

Most candidates set up correct hypotheses, but marks were lost by some candidates through carelessness (e.g. stating $H_0 = 46$ rather than H_0: $\mu = 46$). Some candidates incorrectly chose a one-tail test. For these candidates some follow through marks were subsequently available; however, much inconsistency was seen - for example some candidates set up one-tail tests but followed on with working that was relevant to a two-tail test and vice-versa. Part **(ii)** required not only a correct conclusion, but also evidence of a method which led to that conclusion. It was, therefore, important that candidates showed their comparison of -1.729 with the critical value that they had found (or equivalent comparison of probabilities). Successful candidates often illustrated this comparison diagrammatically.

Answers: **(i)** H_0: $\mu = 46$, H_1: $\mu \neq 46$; **(ii)** No significant difference in times.

Question 2

Part **(i)** of this question was reasonably well attempted, although there was some confusion between variance and standard deviation shown. Unfortunately part **(ii)** was not well attempted at all, with most candidates not appreciating what the question was asking. A large number of candidates did not even give a distribution for their answer (even though they may have given their answer to part **(i)** as N(μ, $\frac{\sigma^2}{n}$)), but merely talked about good estimates or poor estimates of the mean and variance. Many of those who did give a distribution for their answer, did not seem to realise that the Central Limit theorem was being tested and many talked about approximating a distribution. Some candidates did mention a normal distribution for part **(ii)(a)** but very few candidates made a correct statement for part **(ii)(b)**. It was disappointing that even very good candidates showed a lack of understanding of the application of the Central Limit theorem.

Answers: **(i)** μ, $\frac{\sigma^2}{n}$; **(ii)(a)** normal, **(b)** unknown, or normal if the population is normal.

Question 3

Many candidates were able to successfully calculate the 97% confidence interval. Errors included incorrect z-values, with 1.882 being the most frequently seen error, but 0.97 or 0.985 were also seen within the formula instead of a z-value, and even a ϕ-value (0.8378) was commonly seen. Some candidates used 203 rather than *p* in their formula.

Many candidates did not realise what was required in part **(ii)**, with comments on the number of people rather than, as required, the *type* of person in the shopping centre on a Thursday being frequently made. To merely state 'it is not a random sample' was not sufficient and needed a further comment as to why.

Answers: **(i)** (0.672, 0.788); **(ii)** mainly unemployed, retired, or mothers with children i.e. not representative of the whole population.

Question 4

Most candidates used a Poisson distribution in part **(i)**, and the correct parameter of 7.8 was used in many cases. However, a common error made was to calculate $P(X = 3 \mid \lambda = 3.6) + P(X = 3 \mid \lambda = 4.2)$, though a few candidates used the two separate distributions and combined them successfully to find the required $P(X = 3)$, though this was obviously a lengthy method.

Part **(ii)** was also well attempted, though not all candidates applied a correct continuity correction, and not all candidates used the correct variance.

However, despite these common errors, this was, overall, a well attempted question.

Answers: **(i)** 0.0324; **(ii)** 0.215.

Question 5

Candidates found the calculation of the variances in this question particularly difficult.

Whilst some fully correct solutions were seen in part **(i)**, there were also candidates who had problems finding the mean and variance of the required distribution, or even identifying which distribution was required.

Part **(ii)** caused particular problems; many correct means were seen, but very few correct variances. Most candidates were able to standardise in part **(iii)** albeit with incorrect parameters.

Answers: **(i)** 0.0349; **(ii)** 99.5, 113.4; **(iii)** 0.879.

Question 6

Examiners noted many inconsistencies in candidates' solutions to this question. Even those who correctly used a Poisson distribution made errors such as only calculating $P(X = 5)$, or $P(X \varnothing 5)$. Even those who found a sensible tail probability were often unable to make, or even show, a correct comparison. It was also noted by Examiners that, on occasions, comparisons of Poisson probabilities with normal distribution values were seen. In general much confusion was shown.

In part **(iii)** some candidates merely quoted a definition of a Type II error and did not attempt a calculation. When calculations were attempted they were reasonably well completed (though attempts using a normal distribution were seen), with common errors being to use $\lambda = 1.4$ or to calculate $1 - P(X < 4)$ rather than $P(X < 4)$.

Answers: **(i)** Ploughing has increased the number of metal pieces found; **(ii)** No significant increase at the 5% level; **(iii)** 0.395.

Question 7

Some candidates were unable to explain the inequalities in part **(i)** in the context of the question, and merely quoted the information as given, or explained the inequalities in isolation. Part **(ii)** was well attempted by the majority of candidates, as was part **(iii)**, though the usual confusion between mean and median was seen. Parts **(iv)** and **(v)** were not well attempted, with many candidates unable to set up the required equality/inequality, and attempts at integration at this stage were often seen.

Answers: **(i)** All cars stayed between 1 and 9 hours; **(iii)** 3 hours; **(iv)** Greater than 1.39 hours; **(v)** 0.774.